***E4732 CaseStudy 1 name: Yunfei Yan UNI: yy2516***

*(1) Description of coding environment.*

*I use Mobaxterm to code this case study 1 @clic-lab.cs.columbia.edu in linux environment, my compiler is git and my editor is vim.*

*I haven’t used any external library to do the math calculation.*

*(2) My answers to the question:*

*S=100, K=90, r=0.0025, q=0.0125, volatility=0.5, tau=1.*

*Price of Black Scholes Formula:*

***14.448308***

*Price of Finite Difference Methods in* ***2nd order*** *approximation for* ***delta, gamma*** *and* ***Neumann Boundary Condition*** *by tridiagonal matrix solver:*

***14.438311***

*Price of Finite Difference Methods in* ***3rd order*** *approximation for* ***gamma*** *and* ***Neumann Boundary*** *and* ***4th******order*** *approximation for* ***delta*** *by pentadiagonal matrix solver:*

***14.438298***

*Some induction and results about pentadiagonal matrix solver:*

**

*Let: , and use central approximation.*

*We yield the first pair Neumann Boundary Condition:*

**

*Then, we let: , and use forward approximation:*

**

*We yield the second pair Neumann Boundary Condition:*

*, remove  and replace  in the first pair Neumann Boundary Condition, we have:*

**

**

*Similarly*

*,remove , and replace  in the first pair Neumann Boundary Condition, we have:*

**

**

*Finally, we can replace the in the difference equation when , and replace the  in the difference equation when , and get the following result:*

**

*(3) Compare and Conclusion:*

*a.*

*Theoretically, the speed of f3 should be faster than the f5. Due to the machine that I used is the one in Columbia clic-lab, which is very powerful, I didn’t feel the big difference about speed between these two methods.*

*b.*

*Some personal definition about solution:*

***difference:*** *prices acquired by finite difference methods - prices acquired by Black-Scholes formula*

*I have calculated the Black-Scholes price for all the initial S on my grid and plot the difference both for F3 and F5* ***in the attached Excel,*** *and I put the plot here:*

*We can discover several points from the graph:*

*(i) Both methods deviate from the Black-Scholes formula price heavily with the increase of the initial price of underlying S.*

*(ii) Both methods are almost exactly the same with the Black-Scholes price, however, there exists a little “wave” phenomenon* ***which can be ignored*** *at the beginning.*

*(iii) Generally, the curve of f5 is above the curve of f3, which means that the f5 is more consistent with the Black Scholes price in a global version and the accuracy of finite difference method decreases as the S approaches the Smax area.*

*(iiii) The prices yielded from finite difference method would be lower than the Black-Scholes prices in a global vision.*

*(4) Description of the logic present in the written source code*

*In f3.cc, I have written the* ***main function*** *and* ***tridiagonalsolver function****. I initialed all the variables in the main and called the tridiagonalsolver in the main body. ( a little difference with others is that I have already modified the d array before I passed the l array, d array, u array and v array into the tridiagonalsolver which would improve the speed of calculation. )*

*In f5.cc, I have written the* ***main function*** *and* ***pentadiagonalsolver function****. I initialed all the variables in the main function and called pentadiagonalsolver in the main body. In pentadiagonalsolver, I use malloc for all k, l, d, u, m, and v which would not change the value of these origin arrays.*

*I download my code from Mobaxterm directly with .cc type. You can open it using notepad ( I have already tried, and it works. ) If you cannot open it, please contact me.*

*Thank you!*